Undulatory instability of the nematic-isotropic interface

V. A. Raghunathan
Centre de Recherche Paul Pascal, Avenue Schweitzer, 33600 Pessac, France
(Received 21 July 1994)

A linear stability analysis of the undulatory instability of the nematic-isotropic (*N-I*) interface under a vertical magnetic field is presented. It is shown that the boundary conditions on the lower surface of the nematic layer have a very significant influence on the threshold characteristics. Furthermore, in the case of homoetropic alignment, the interface is found to be spontaneously deformed if the thickness of the nematic layer is less than a critical value.

PACS number(s): 61.30.Gd

I. INTRODUCTION

The interface between a nematic and an isotropic liquid is characterized by a preferential ordering of the director $\hat{\mathbf{n}}$, which describes the orientational ordering in the nematic medium. Due to the symmetry in the plane of the interface, only the angle between $\hat{\mathbf{n}}$ and the local normal to the director is fixed and the azimuthal orientation of $\hat{\mathbf{n}}$ is not restricted. As the nematic medium is anisotropic, it follows that a planar interface becomes unstable under a sufficiently strong electric or magnetic field if the alignment imposed by the interface is not along a direction favored by the field. The tilt of the interface from the horizontal leads to a more favorable orientation of $\hat{\mathbf{n}}$ with respect to the field direction, thus lowering the free energy. Furthermore, as the interfacial tension and the density difference between these phases of a nematogenic material are very small, relatively weak fields can destabilize the interface.

There are two possible modes of instability of the*N-I* interface under an applied electric or magnetic field. The first is the de Gennes instability [1,2], where the interface splits up into domains of opposite tilt from the horizontal above a critical value of the field. The azimuthal angle of $\hat{\mathbf{n}}$ at the interface jumps by π across the boundary between two domains, thus creating surface disclinations in the director field. Such interfacial deformations have been experimentally observed [3,4]. The second is the instability observed by Yokoyama, Kobayashi, and Kamei [5], where the interface becomes undulatory above a threshold field, with the wave vector of the undulations oriented normal to the plane containing the initial director field. The onset of the interfacial deformation is accompanied by a periodic twist distortion in the nematic layer due to a periodic change in the azimuthal angle of $\hat{\mathbf{n}}$ at the interface. As discussed in Ref. [5], this twist distortion along with the interfacial undulation leads to a more favorable orientation of the nematic director near the interface with respect to the vertical field. Consequently, the planar interface becomes unstable if the field is sufficiently strong. As the orientation of $\hat{\mathbf{n}}$ varies smoothly across the interface, no surface disclinations are created in this case.

In order to understand the origin of the undulatory instability, consider an *N-I* interface lying in the x-y plane, subjected to a vertical magnetic field H. It may be noted here that in the case of an electric field the nonuniformity of the field in the nematic layer has to be taken into account, but the results are not expected to be qualitatively different. The nematic director is only restricted to lie on a cone of easy alignment directions at the interface. This degeneracy can be lifted by orienting the director along the y axis at the lower surface of the nematic layer, so that $\hat{\mathbf{n}}=\hat{\mathbf{n}}(y,z)$. Furthermore, the nematic is assumed to have a positive diamagnetic anisotropy ($\Delta\chi$) and consequently, $\hat{\mathbf{n}}$ tends to align along H. As H is increased, the distortions in the director field become confined to two thin regions close to the two surfaces of the nematic layer. Now let the interface be tilted from the horizontal about the y axis. As shown in Fig. 1, by changing the orientation of the director on the cone of easy directions, we can align it more favorably with respect to H and hence decrease the magnetic energy of the system. This reorientation of $\hat{\mathbf{n}}$ near the interface introduces a twist in the nematic layer, with the sign of the twist determined by the sign of the interfacial tilt. On the other hand, it can decrease the amount of splay-bend distortion as the director in the bulk is aligned more favorably with respect to H. Furthermore, the tilt of the interface increases the surface energy due to the interfacial tension and the gravitational energy arising from the difference in the densities of the two phases. The latter forces the interfacial deformation to be periodic with alternating regions of opposite tilt. The threshold field is determined by the balance between the changes in the bulk and surface energies of the system.

In this paper, a linear stability analysis of the undulatory instability is presented. The *N-I* interface can be produced in an experiment by creating a temperature gradient across the thickness of the sandwich cell containing the liquid. As demonstrated in Ref. [5], it can also be obtained at a fixed temperature by making the wetting properties of the two surfaces of the cell different. The interface is then produced by maintaining a weakly doped material at a temperature within the range of coexistence of the two phases. In all the discussion below only the latter situation is considered, and hence the material parameters are not temperature dependent. The basic equa-

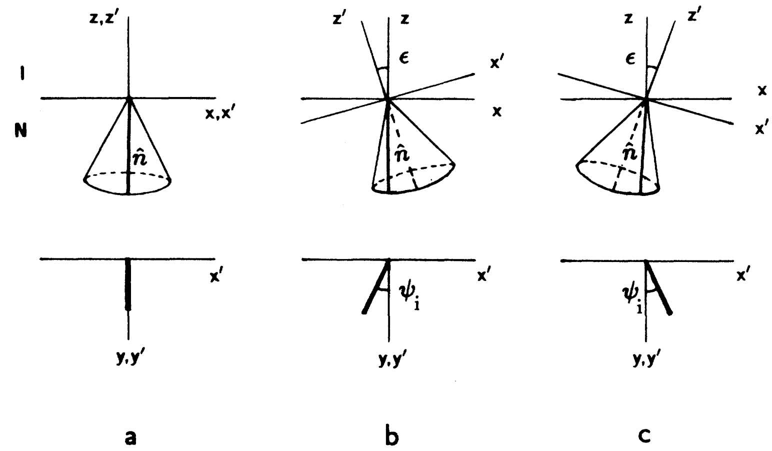

FIG. 1. A schematic representation of the coupling between interfacial tilt and twist deformation in the nematic layer close to the N-I interface, responsible for the undulatory instability. The director at the interface is only restricted to lie on a cone of easy alignment directions. The applied field is along the z axis. Below the threshold the director is in the y-z plane (a). Above the threshold the interface is tilted by an angle $\epsilon=(dh/dx)$ from the horizontal and the director at the interface is twisted by an angle ψ_i about the local normal [(b) and (c)]. Note that the sign of ψ_i depends on that of ϵ.

tions are derived in Sec. II and some analytical results, obtained under certain approximations, are described in Sec. III. The conclusions reached here are confirmed by the rigorous numerical treatment of the problem discussed in Sec. IV. The results of the qualitative analysis of the instability presented in Ref. [5] are also discussed in this section, in light of the present study.

II. THEORY

Consider a nematic layer of thickness d sandwiched between the isotropic phase and a solid substrate. Let $\delta(z)$ be the angle between $\hat{\mathbf{n}}$ and the z axis at subcritical values of the applied field. At the interface, $\delta(z=d)=\delta_i$. Due to the finite anchoring strength of the nematic director at the interface, δ_i is a function of H. On the lower surface of the layer $\hat{\mathbf{n}}$ is assumed to be oriented either along the z axis (homeotropic alignment) or along the y axis (planar alignment) with infinite anchoring strength. Thus at subcritical values of H the director field in the nematic is given by

$$\hat{\mathbf{n}}=(0,\sin\delta,\cos\delta)\ .$$

In order to study the stability of the interface, let us consider a perturbation of the interface from the horizontal of the form,

$$h=h_0\cos qx\ . \tag{1}$$

Here the wave vector of the perturbation is assumed to be along x, in agreement with the experimental observations. Hence all the variables in the problem are independent of y. It may be noted here that an interfacial deformation with q along y corresponds to the de Gennes instability and associated surface disclination lines. Let $\phi(x,z)$ describe the twist deformation created in the nematic layer above the instability threshold. As discussed earlier, this twist along with the interfacial undulation $h(x)$ changes $\delta(z)$, the angle between $\hat{\mathbf{n}}$ and the z axis. Let $\theta(x,z)$ describe this change. Then above the threshold the director field is given by

$$\begin{aligned} n_x&=\sin(\delta+\theta)\sin\phi\ ,\\ n_y&=\sin(\delta+\theta)\cos\phi\ ,\\ n_z&=\cos(\delta+\theta)\ . \end{aligned} \tag{2}$$

Let $x'y'z'$ be the local coordinate system at the interface, tilted about the y axis by an angle $\epsilon=(dh/dx)$ with respect to the laboratory xyz system. Let ψ_i be the twist at the interface in the $x'y'z'$ system. Clearly the angle between the z' axis and $\hat{\mathbf{n}}$ at the interface is $\delta_i(H)$ even above the threshold. Therefore, the director at the interface is given by

$$\begin{aligned} n_{x'}&=\sin\delta_i\sin\psi_i\ ,\\ n_{y'}&=\sin\delta_i\cos\psi_i\ ,\\ n_{z'}&=\cos\delta_i\ . \end{aligned}$$

Transforming these to the xyz system, we get

$$\begin{aligned} n_x&=n_{x'}\cos\epsilon-n_{z'}\sin\epsilon\ ,\\ n_y&=n_{y'}\ ,\\ n_z&=n_{z'}\cos\epsilon+n_{x'}\sin\epsilon\ . \end{aligned} \tag{3}$$

At the interface expressions (2) and (3) must be equivalent. Equating the two sets and neglecting the higher-order terms, we find

$$\phi_i=-\epsilon\cot\delta_i+\psi_i\ , \tag{4}$$

$$\theta_i=\tfrac{1}{2}\epsilon^2\cot\delta_i-\epsilon\psi_i\ , \tag{5}$$

where the subscript i denotes the value at the interface. The first term in each of the above two expressions gives the contribution arising purely from the tilt of the interface. It is clear from Eq. (5) that this contribution to θ_i is always positive. Since $\Delta\chi$ is positive, the second term, which is a combination of the surface tilt and the twist is, therefore, essential for the onset of the instability. The values of ϕ and θ in the bulk of the nematic are determined by their values at the interface. Therefore, their x dependences in the bulk can be assumed to be the same as those at the interface. From Eqs. (1), (4), and (5) it follows that

$$\phi(x,z)=\phi(z)\sin qx\ , \quad (6)$$

$$\theta(x,z)=\theta(z)\sin^2 qx\ . \quad (7)$$

The bulk free energy density of the nematic is given by

$$F=\tfrac{1}{2}[K_1(\mathrm{div}\hat{\mathbf{n}})^2+K_2(\hat{\mathbf{n}}\cdot\mathrm{curl}\hat{\mathbf{n}})^2+K_3(\hat{\mathbf{n}}\times\mathrm{curl}\hat{\mathbf{n}})^2 -\Delta\chi(\hat{\mathbf{n}}\cdot\mathbf{H})^2]\ ,$$

where K_1, K_2, and K_3 are the splay, twist, and bend elastic constants, respectively. Substituting for $\hat{\mathbf{n}}$ from Eq. (2) and neglecting terms independent of ϕ and θ, the following expression for the change in the bulk free energy density of the nematic above the threshold of the instability is obtained.

$$\Delta F=\tfrac{1}{2}K_1\sin^2\delta\left[\frac{\partial\phi}{\partial x}\right]^2+(K_1\sin^2\delta+K_3\cos^2\delta)\left[\frac{\partial\delta}{\partial z}\right]\left[\frac{\partial\theta}{\partial z}\right]-K_1\sin^2\delta\left[\frac{\partial\delta}{\partial z}\right]\left[\frac{\partial\phi}{\partial x}\right] +\tfrac{1}{2}(K_2\sin^2\delta+K_3\cos^2\delta)\sin^2\delta\left[\frac{\partial\phi}{\partial z}\right]^2+\tfrac{1}{2}(K_1-K_3)\sin2\delta\left[\frac{\partial\delta}{\partial z}\right]^2\theta+\tfrac{1}{2}\Delta\chi H^2\sin2\delta\,\theta\ . \quad (8)$$

In this expression only the lowest-order terms have been retained as we are interested in the behavior of the system close to the instability threshold.

Minimizing ΔF with respect to ϕ and θ, the following differential equations for δ, ϕ, and θ are obtained. Near the threshold of the instability most of the deformation in the director field is confined to a narrow layer close to the interface. The thickness of this layer is typically equal to the magnetic coherence length, and can be much smaller than the thickness of the nematic layer. In these equations the x dependence of ϕ and θ have been neglected as the wavelength of the interfacial undulation can be expected to be very much larger than the length over which the director field is deformed, as indeed confirmed by the calculations.

$$(K_1\sin^2\delta+K_3\cos^2\delta)\left[\frac{\partial^2\delta}{\partial z^2}\right]+\tfrac{1}{2}(K_1-K_3)\sin2\delta\left[\frac{\partial\delta}{\partial z}\right]^2-\tfrac{1}{2}\Delta\chi H^2\sin2\delta=0\ , \quad (9)$$

$$\frac{\partial}{\partial z}\left[(K_2\sin^2\delta+K_3\cos^2\delta)\sin^2\delta\left[\frac{\partial\phi}{\partial z}\right]\right]=0\ , \quad (10)$$

$$(K_1\sin^2\delta+K_3\cos^2\delta)\left[\frac{\partial^2\theta}{\partial z^2}\right]+(K_1-K_3)\sin2\delta\left[\frac{\partial\delta}{\partial z}\right]\left[\frac{\partial\theta}{\partial z}\right] +(K_1-K_3)\left[\sin2\delta\left[\frac{\partial^2\delta}{\partial z^2}\right]+\cos2\delta\left[\frac{\partial\delta}{\partial z}\right]^2\right]\theta-\Delta\chi H^2\cos2\delta\,\theta=0\ . \quad (11)$$

The boundary conditions at $z=0$ are

$$(\delta,\theta,\phi)=\begin{cases}(0,0,\phi_i)\ , & \text{homeotropic alignment}\\ (\pi/2,0,0)\ , & \text{planar alignment}\ .\end{cases}$$

And those at $z=d$ are

$$(W/2)\sin2(\delta_0-\delta_i)-(K_1\sin^2\delta_i+K_3\cos^2\delta_i)\left[\frac{\partial\delta}{\partial z}\right]_i=0\ ,$$

$$\theta=\theta_i \text{ and } \phi=\phi_i\ ,$$

where W is the anchoring strength of the director at the interface and δ_0 the angle made by $\hat{\mathbf{n}}$ with the z' axis at the interface in the absence of any surface torques. ϕ_i and θ_i are given by Eqs. (4) and (5), respectively.

The increase in the surface free energy density above the threshold is given by

$$F_s=\tfrac{1}{2}\Delta\rho g h^2+\tfrac{1}{2}\gamma\epsilon^2\ ,$$

where $\Delta\rho$ is the density difference between the nematic and the isotropic phases, γ the interfacial tension, and g the acceleration due to gravity. Then the change in the total free energy of the system per wavelength (λ) of the interfacial undulation can be written as

$$f_t=\int_0^{\lambda}\left[\int_0^{d+h}\Delta F\,dz+F_s\right]dx\ .$$

III. ANALYTICAL RESULTS

A. Homeotropic alignment

The homeotropic alignment at the lower surface precludes any twist distortion across the thickness of the nematic layer. Furthermore, in this case it is possible to solve Eqs. (9)–(11) analytically in the one elastic constant approximation and in the limit of small δ_0 and strong anchoring of the director at the interface. The solutions are

$$\delta=\frac{\delta_i}{\sinh(w)}\sinh(w\xi) , \qquad (12)$$

$$\theta=\frac{\theta_i}{\sinh(w)}\sinh(w\xi) , \qquad (13)$$

and

$$\phi=\phi_i , \qquad (14)$$

where $w=(\Delta\chi/K)^{1/2}Hd$ and $\xi=z/d$. Substituting these solutions in Eq. (8) the change in the bulk free energy density of the system due to the interfacial deformations is obtained. Integrating this expression over the thickness of the nematic layer and over one wavelength of the deformation, we get

$$f_b=\frac{\pi Kqd}{8w\sinh^2 w}\{[\sinh(2w)-2w]\delta_i^2\phi_i^2 + 4(w/qd)^2\sinh(2w)\delta_i\theta_i\} .$$

Substituting for θ_i and ϕ_i from Eqs. (4) and (5), and minimizing the resulting expression with respect to ψ_i, we get

$$\psi_i=-\frac{1}{\delta_i}\left[q+\frac{2w^2}{qd^2}\frac{\sinh(2w)}{\sinh(2w)-2w}\right]h_0 .$$

Using this expression we find the total change in the free energy of the system per one wavelength of the interfacial deformation to be

$$f_t=\frac{\pi}{2}(\gamma_{\text{eff}}q+\Delta\rho_{\text{eff}}g/q)h_0^2 ,$$

where the effective surface tension,

$$\gamma_{\text{eff}}=\gamma-K(w/d)\coth w , \qquad (15)$$

the effective density difference,

$$\Delta\rho_{\text{eff}}=\Delta\rho-(4K/g)(w/d)^3[\cosh^2 w/(\sinh 2w-2w)] , \qquad (16)$$

and K is the elastic constant. Thus the change in the bulk free energy of the nematic layer due to the reorientation of $\hat{\mathbf{n}}$ associated with the interfacial deformations effectively lower the interfacial tension and the density difference between the two phases. Using typical values of the material parameters and the threshold field, the second of these contributions is found to be a few orders of magnitude larger than the first. Hence the interface becomes unstable when the applied field is large enough to make $\Delta\rho_{\text{eff}}$ zero. The instability develops with a threshold wave vector $q_{\text{th}}=0$, as this term is proportional to q^{-1}. The threshold field can be calculated from the condition that $\Delta\rho_{\text{eff}}=0$. For $\Delta\rho=2.0\times10^{-3}$ erg/cm^3, $K=3.0\times10^{-7}$ dyn, and $d=100$ μm, this gives a threshold field of 190 G. Interestingly, these contributions from the bulk free energy do not vanish when $H=0$. Thus in the limit of zero field, Eq. (16) reduces to

$$\Delta\rho_{\text{eff}}=\Delta\rho-(3K/g)/d^3 .$$

Therefore, if d is less than a critical value given by

$$d_c^3=3K/(\Delta\rho g) ,$$

the interface is intrinsically unstable. For the above mentioned values of K and $\Delta\rho$, d_c turns out to be 77 μm. For thinner samples, the change in the director profile accompanying the interfacial undulation reduces the overall elastic distortion in the nematic layer, leading to a spontaneous deformation of the interface. Such spontaneous interfacial deformation is also expected in the case of the de Gennes instability [3,6]. These conclusions are borne out by the results of the numerical calculations presented in the following section.

B. Planar alignment

In the case of planar alignment the periodic variation of ψ_i at the interface introduces a twist deformation across the thickness of the layer. This additional deformation makes the threshold behavior very different from that in the earlier case. Though it is not possible to solve Eqs. (9)–(11) analytically we can still get some insight into the threshold behavior if we assume that the solutions have the form: $\theta(z)=\theta_i f_1(z)$ and $\phi(z)=\phi_i f_2(z)$, where f_1 and f_2 are independent of θ_i and ϕ_i. Following the same procedure as above, the change in the bulk free energy per wavelength of the deformation is found to be

$$f_b=-\frac{\pi K}{8d}\left[\frac{(w^2I_2+2I_3)^2q}{I_4+I_1(dq)^2}+2(w^2I_2 + 2I_3)\cot(\delta_i)q\right]h_0^2 ,$$

where $I_1=\int_0^1\sin^2\delta f_2^2 d\xi$, $I_2=\int_0^1\sin 2\delta f_1 d\xi$, $I_3=\int_0^1(d\delta/d\xi)(df_1/d\xi)d\xi$ and $I_4=\int_0^1\sin^2\delta(df_2/d\xi)^2 d\xi$. The first term in the above expression is proportional to H^4, while the second is proportional to H^2. Moreover, in comparison with the homeotropic case, the former can be expected to be the dominant one, and we see that it has a minimum at a nonzero value of q due to the twist deformation in the nematic layer. Hence we can anticipate the instability to set in with $q_{\text{th}}\neq0$ as indeed confirmed by the numerical calculations.

IV. NUMERICAL RESULTS

In order to determine the threshold, a small value of $h_0 \ll d$ is first selected. Then for a given value of H Eq. (9) is solved to obtain $\delta(z)$. Equations (10) and (11) are subsequently solved to find $\theta(z)$ and $\phi(z)$ for given values of q and ψ_i. These differential equations were solved using finite difference methods [7]. Using the x dependences of h, ϕ_i, and θ_i, given by Eqs. (1), (4), and (5), the change in the total free energy per one wavelength of the surface undulation f_t is then calculated and minimized with respect to ψ_i. Thus for given values of q and H, the minimum of f_t is found. The value of H is then varied to make $\min f_t = 0$; this is the critical value H_c for the onset of the instability for the given value of q. Minimization of H_c with respect to q gives the threshold wave vector $q_{\rm th}$ and the threshold field $H_{\rm th}$. The following values of the material parameters were used in the calculations [4,5,8]. $K_1 = 2.7 \times 10^{-7}$ dyn, $K_2 = 1.4 \times 10^{-7}$ dyn, $K_3 = 3.0 \times 10^{-7}$ dyn, $\Delta\chi = 1.0 \times 10^{-7}$ esu, $\gamma = 2.0 \times 10^{-2}$ erg/cm^2, $\Delta\rho = 2.0 \times 10^{-3}$ g/cm^3, and $W = 1.0 \times 10^{-3}$ erg/cm^2.

A. Homeotropic alignment

The profiles of $\delta(z)$ and $\theta(z)$ across the nematic layer, obtained from the calculations, are shown in Fig. 2 for $d = 100$ μm and $\delta_0 = 1.0$ rad. Note that these two profiles are very similar as one would expect from Eqs. (12) and (13). On the other hand, ϕ is independent of z in the homeotropic case. The variation of the critical field H_c with q obtained from the calculations is shown in Fig. 3. In agreement with the above analysis the interface is found to be spontaneously deformed if the thickness is less than about 80 μm. For thicker samples the instability sets in at a threshold wave vector $q_{\rm th} = 0$, as H is increased above a threshold value. The neglect of the x dependence of the deformation in the nematic layer in Eqs. (10) and (11) is, therefore, fully justified in this case as the wavelength of the interfacial undulations is infinite. The variation of the threshold field $H_{\rm th}$ with δ_0 is shown in Fig. 4. $H_{\rm th}$ is found to increase with the thickness of the nematic layer. As H is increased above the threshold value the interface becomes unstable for all values of q less than a critical value q_c. Figure 5 presents the variation of q_c with H calculated from the linear theory. The variation obtained from the condition $\Delta\rho_{\rm eff} = 0$, is also given for comparison. If the interface is subjected to a magnetic field stronger than $H_{\rm th}$ the interface can be expected to develop deformations corresponding to the fastest growing mode whose wave vector lies between 0 and q_c. However, with time these structures would slowly coarsen and reach the free energy minimum at $q = 0$. Since in the case of homeotropic alignment the wavelength of the deformation is set by the lateral size of the system, the only means to visualize the instability may be to observe the response of the system to a strong field.

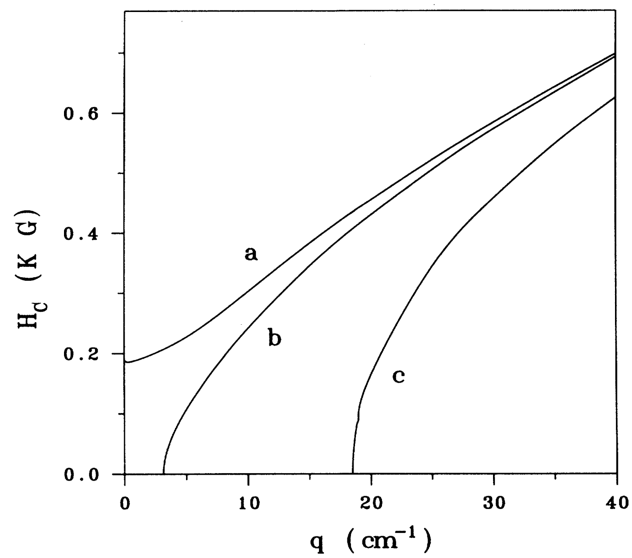

FIG. 3. Variation of the critical field H_c with the wave vector q for homeotropic alignment for $\delta_0 = 1.0$ rad and $d = 100$ μm (a), 80 μm (b), and 50 μm (c).

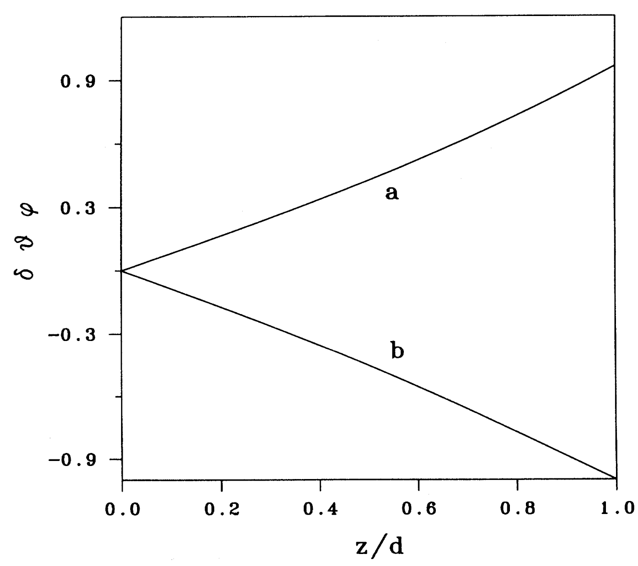

FIG. 2. Profiles of $\delta(z)$ in radians (a) and $\theta(z)/|\theta_i|$ (b) across the nematic layer slightly above the threshold in the case of homeotropic alignment for $\delta_0 = 1.0$ rad and $d = 100$ μm.

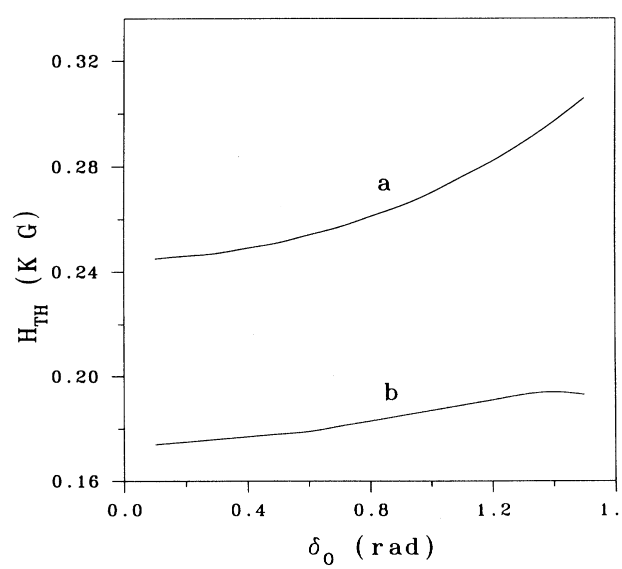

FIG. 4. Variation of the threshold field $H_{\rm th}$ with δ_0 for homeotropic alignment for $d = 200$ μm (a) and 100 μm (b).

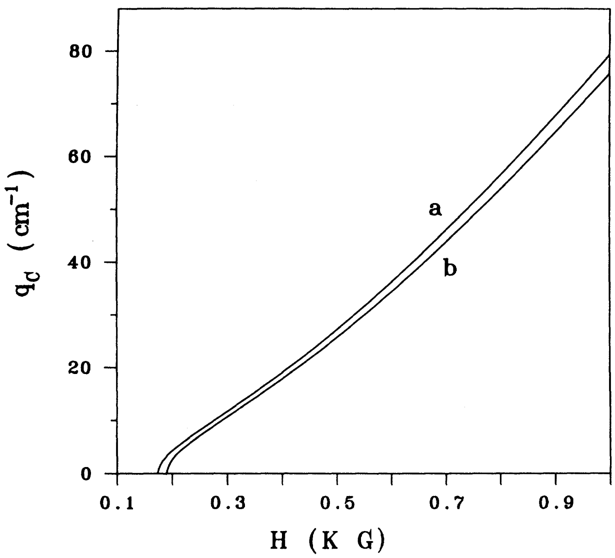

FIG. 5. Dependence of the critical field q_c on H in the case of homeotropic alignment for $d = 100$ μm and $\delta_0 = 0.1$ rad, obtained from the numerical calculations (a) and from the condition $\Delta\rho_{\rm eff} = 0$ (b).

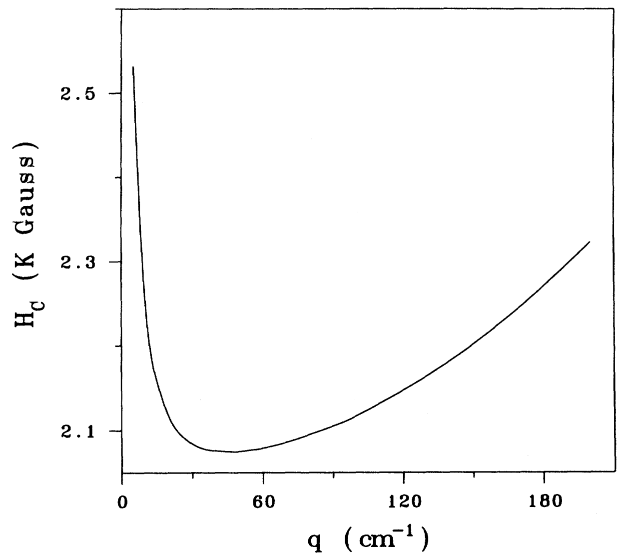

FIG. 7. Variation of the critical field H_c with the wave vector q for planar alignment for $\delta_0 = 1.0$ rad and $d = 50$ μm.

B. Planar alignment

The variation of $\delta(z)$, $\theta(z)$, and $\phi(z)$ across the nematic layer are shown in Fig. 6 for $d = 50$ μm and $\delta_0 = 1.0$ rad. Figure 7 presents the variation of H_c with q for $d = 50$ μm and $\delta_0 = 1.0$. In conformity with the earlier discussion we see that H_c has a minimum at $q_c \approx 50$ cm^{-1}. The variations of $H_{\rm th}$ and $q_{\rm th}$ with δ_0 are given in Figs. 8 and 9, respectively, for a few values of d. From Figs. 6 and 9 it can be seen that the thickness of the strongly deformed region in the nematic layer is typically an order of magnitude smaller than the wavelength of the interfacial undulations. Hence the neglect of the x dependence of the deformation in comparison with its z dependence in Eqs. (10) and (11) is a reasonable approximation. In this case, the threshold field is found to increase with decreasing d. This can be understood as again due to the twist deformation in the sample, which is stronger in thinner layers. Furthermore, it is clear that both $H_{\rm th}$ and $q_{\rm th}$ increase on decreasing d. This trend is in agreement with the experimental observations of Ref. [5] on planar aligned nematic layers. Furthermore for a

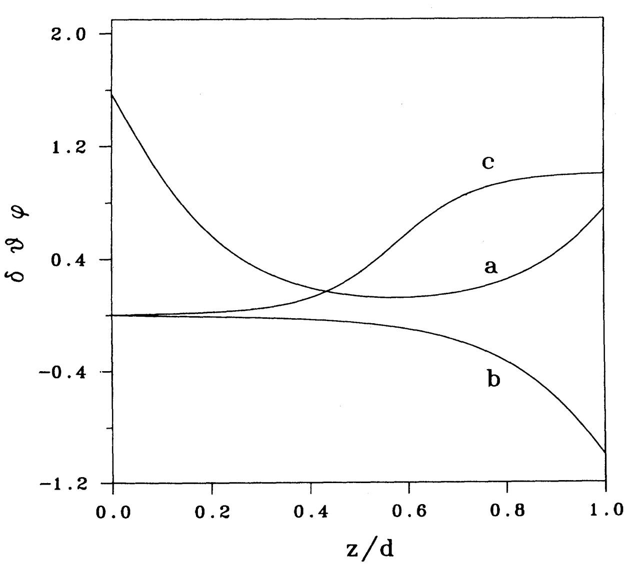

FIG. 6. Profiles of $\delta(z)$ in radians (a), $\theta(z)/|\theta_i|$ (b), and $\phi(z)/\phi_i$ (c) across the nematic layer slightly above the threshold in the case of planar alignment for $\delta_0 = 1.0$ rad and $d = 50$ μm.

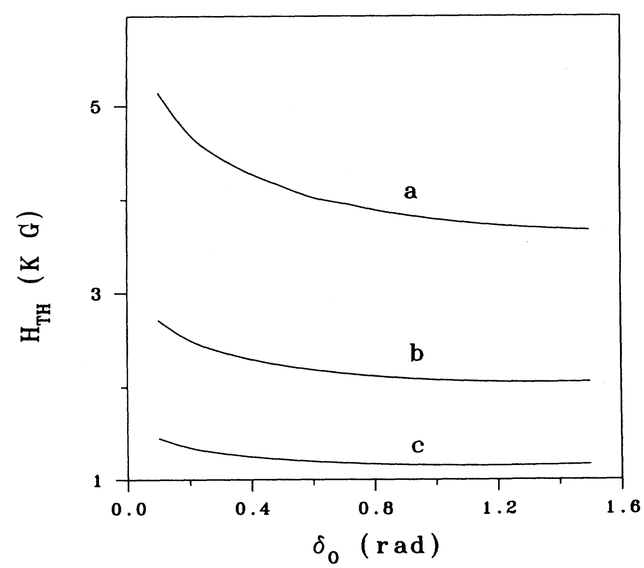

FIG. 8. Variation of the threshold field $H_{\rm th}$ with δ_0 for $d = 25$ μm (a), 50 μm (b), and 100 μm (c).

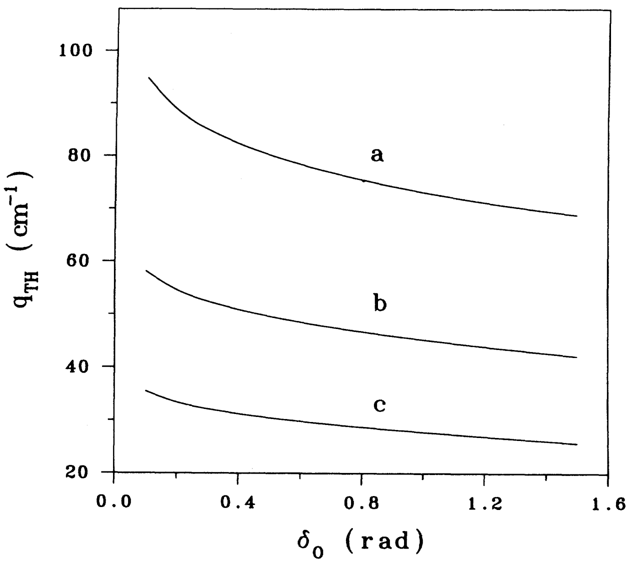

FIG. 9. Variation of the threshold wave vector $q_{\rm th}$ with δ_0 for $d=25$ μm (a), 50 μm (b), and 100 μm (c).

25 μm thick nematic layer with $\delta_0 \approx 1$ rad, they find a critical voltage of about 2.5 V and $q \approx 200$ cm^{-1}. Taking $\Delta\epsilon = 5$ this corresponds to a magnetic field of about 3.9 KG. These values of $H_{\rm th}$ and $q_{\rm th}$ are clearly comparable to those found from the present calculations. However, since not all the material parameters of the nematic used in the experiments are known, it is not possible to make a proper comparison. Furthermore, it may be recalled here that in the case of an electric field the nonuniformity of the field in the nematic layer has to be taken into account and, therefore, a very close agreement between the experimental and calculated values of $H_{\rm th}$ and $q_{\rm th}$ cannot anyway be expected.

A qualitative analysis of the undulatory instability is presented in Ref. [5] along with the experimental results. They assume that near the threshold the director in the bulk of the nematic layer is aligned along the field direction. Thus the boundary conditions on the lower surface of the nematic layer are considered to be irrelevant for the threshold behavior. Neglecting the twist distortion across the nematic layer they come to the wrong conclusion that the instability occurs with $q_{\rm th}=0$ for planar alignment. The observed nonzero value of $q_{\rm th}$ is attributed by them to the fastest growing mode. However, Fig. 6 shows that in the planar case the threshold field is not high enough to align the director in the bulk along z, and so the twist distortion across the thickness of the nematic layer has to be taken into account. This results in the vastly different behaviors in the two cases.

V. CONCLUSION

A linear stability analysis of the undulatory instability of the nematic-isotropic interface under a vertical magnetic field has been presented. The analytical results obtained under certain approximations are confirmed by detailed numerical calculations. The boundary conditions at the lower surface of the nematic layer are found to have a significant influence on the threshold behavior. Thus in the case of planar alignment the instability is found to set in at a nonzero value of the threshold wave vector $q_{\rm th}$, while in the case of homeotropic alignment $q_{\rm th}=0$. Furthermore, in the latter case, the interface is shown to be spontaneously deformed if the thickness of the nematic layer is less than a critical value. The results of the calculations for planar alignment are in broad agreement with the experimental observations of Yokoyama, Kobayashi, and Kamei [5].

ACKNOWLEDGMENTS

Parts of this work were carried out at the Raman Research Institute, Bangalore, India and at Université du Maine, Le Mans, France. I thank N. V. Madhusudana and D. Ausserré for many useful discussions. Financial support from Centre National de la Recherche Scientific, France and from the Indo-French Centre for the Promotion of Advanced Research is gratefully acknowledged.

[1] P. G. de Gennes, *The Physics of Liquid Crystals* (Clarendon, Oxford, 1979).
[2] P. G. de Gennes, Solid State Commun. **48**, 213 (1970).
[3] N. V. Madhusudana and K. R. Sumathy, Mol. Cryst. Liq. Cryst. **92**, 193 (1983).
[4] S. Faetti and V. Palleschi, J. Phys. France **46**, 415 (1985).
[5] H. Yokoyama, S. Kobayashi, and H. Kamei, Mol. Cryst. Liq. Cryst. **129**, 109 (1985).
[6] S. Chandrasekhar and G. S. Ranganath, Adv. Phys. **35**, 507 (1986).
[7] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes* (Cambridge University Press, Cambridge, 1986).
[8] N. V. Madhusudana and R. Pratibha, Mol. Cryst. Liq. Cryst. **89**, 249 (1982).